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Volume Uncertainty in Construction Projects: A Real Options Approach

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Abstract

This paper proposes a model aiming to quantify the impact that a specific type of uncertainty -volume uncertainty- may produce on construction projects' value and on the optimal bid price, in the context of bidding competitions. Volume uncertainty is present in most construction projects since managers do not know, during the bid preparation stage, the exact volume of work that will be executed during the project's life cycle. Volume uncertainty leads to profit uncertainty and hence the model integrates a discrete-time stochastic variable, designated as "additional value", i.e., the value that does not directly derive from the execution of the tasks specified in the bid documents, and which can only be properly quantified by undertaking an incremental investment in human capital and technology. The model determines that, even only recurring to the skills of their own experienced staff, contractors will produce a more competitive bid provided that the expected amount for the additional profit is greater than zero. However, construction managers often need to hire specialized firms and highly skilled professionals in order to quantify the expected amount of additional value and, hence, the impact of such additional value in the optimal bidding price. Based on the option to sign the contract and to perform the project by the selected bidder, identified and evaluated by Ribeiro et al. (2017), the model's outcome is the threshold value for this incremental investment. A decision rule is then reached: construction managers should invest in human capital and technology provided that the cost of such incremental investment does not exceed the predetermined threshold value.

Keywords: construction projects, real options, uncertainty, flexibility, price determination.

JEL Classification Codes: G31, D81.

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1 INTRODUCTION

Uncertainty surrounding construction projects is a crucial element that should be adequately managed since it may have a considerable impact on the construction company's overall performance. Construction companies or contractors are firms operating in the construction industry whose business resides in executing a set of tasks previously established by the client.[1] The amount of tasks to be performed constitutes a project, job or work. The vast majority of projects in the construction industry are assigned through what is known as "tender" or "bidding" processes (Christodoulou (2010); Drew et al. (2001)), this being the most popular form of price determination (Liu and Ling (2005); Li and Love (1999)). In a tender or bidding process, a certain number of contractors (bidders) compete to execute a project by submitting a single-sealed proposal until a specific date previously defined by the client. Potential bidders have access to a what is commonly known as the "bid package". This package contains a set of technical pieces (often also referred as "tender documents") which serve as the basis for establishing the price to include in the bid proposal. More specifically, the package includes plans and technical drawings, a proposal form, the "general conditions" covering procedures which are common to all construction contracts and the "special conditions" containing the procedures to be used and that are unique to the project in question (Halpin and Senior (2011)), including information about the type of contract that will be enforced. The usual format of a tender or bidding process is based on the rule that - all other things being equal - the contract will be awarded to the competitor that submitted the lowest price (Christodoulou (2010); Cheung et al. (2008); Chapman et al. (2000)) or, which is the same, the lowest bid. Thus, the bidder proposing the lowest price will most likely be invited to sign the contract and, if the contract is actually signed, he or she will have to invest a substantial amount of money by incurring in the necessary direct costs to execute the project, *i.e.*, the construction costs.

Traditionally, construction management literature has been placing more emphasis on the negative effects which means researchers seem to be more concerned with ways to deal with the risks involving the construction activities and how they may affect the value of the project through a negative impact on the construction costs. In fact, few authors have been addressing uncertainty as a source of opportunity, as it is the case of Ford et al. (2002), Ng and Bjornsson (2004) and Yiu and Tam (2006). Ford et al. (2002) argued that construction projects may include specific sources of uncertainty that affect project value, but not necessarily just by reducing it. This argument is supported by Ng and Bjorns-

[1] For the purposes of this research, we exclude those situations where contractors execute their own projects, as it is often the case of construction companies operating in the real estate sector.

son (2004) when they state that, even though uncertainty can lead to cost over-runs and delays, it can also produce positive return if properly managed. Following this line of thought, applied the real options approach to evaluate the intrinsic value of uncertainty and the managerial flexibility deriving from the options to defer and to switch modes of construction. Therefore, uncertainty can also be seen as a source of opportunities, rather than just an element that may cause undesirable effects during the construction stage - in clear opposition to the traditional view that ”all uncertainty presumes loss” (Mak and Picken (2000)). Ford et al. (2002) acknowledge the fact that many construction project conditions evolve over time and, thus, managerial choices for effective decision-making cannot be completely and accurately determined during the pre-project planning period. In fact, these authors observe that many aspects of construction projects are uncertain, such as input prices, the weather conditions, the length of some activities and the overall duration of the project, among others, meaning that the effects of some of these sources of uncertainty can only be recognized and properly managed as the project unfolds. This argument is also supported by Mattar and Cheah (2006) when they mention that contractors typically learn more about the value of the project as they invest over time and uncertainties are resolved. Even though we recognize the fact that the possible consequences of some sources of uncertainty cannot be anticipated, we do believe that others can be predicted and accounted for during the bid preparation process. Moreover, we will try to demonstrate that it is possible to establish a support decision model that accommodates the expected impact of a specific type of uncertainty - the uncertainty associated with the amount of work to be executed during the project's life cycle - on the project value. The model we propose in the present paper builds on this crucial aspect by focusing on a specific source of uncertainty which may lead to a greater project value by increasing the expected amount of work to be executed during the construction stage. This means we believe that this source of uncertainty is - at least - as decisive as the others in adding value to the project. Threfore, managers should recognize its importance by planning and strategically managing this element in a way that improves the project value and, as we will demonstrate, their competitiveness in a bidding competition context.

Despite the fact that project value can be substantially increased by reducing costs, we would like to reinforce the idea that project value can also be increased by raising more income. As we will see, more income means the income that is generated through actually executing, during the construction phase, a certain amount of tasks which were not included in the tender documents. We are thus concerned with the uncertainty that may lead to more project value by increasing the amount of work to be performed by contractors, “vis-a-vis” with the amount of work contractors are contractually bound to execute.

We will refer to this type of value as “hidden-value”.[2]

Hidden-value should be captured and quantified in the pre-project stage while the bid proposal is being prepared, by carefully analyzing the portions of the project where it may be concealed. Ford et al. (2002) observed that hidden-value is present in the most uncertain portions of the project, enabling us to sustain that skilled engineers and experienced managers - whose responsibility is to prepare the bid proposal - have a fairly good knowledge, based on their accumulated experience, of “where to look for”. Chapman et al. (2000) stated that the bid preparation process begins with a preliminary assessment of the tender documents. We sympathize with this statement and argue that, in this preliminary assessment, it is possible to recognize and quantify hidden-value and, more specifically, to stipulate a high-estimate and a low-estimate to this hidden-value and to attribute a probability of occurrence to each of the estimates just by undertaking a preliminary analysis of the tender documents.[3] However, the quantification of hidden-value with accuracy is, in most practical situations, a goal that can only be achieved by performing an exhaustive investigation of all the bid documents and performing studies that often require the use of specific advanced technologies, which many contractors do not possess. Therefore, many construction companies will have to invest in human capital and technology by recurring to external firms with the purpose of supplying managers with more accurate information concerning the project in hands, as Kululanga et al. (2001) stated. [4] In fact, these authors argue that an awareness of job factors, which may give rise for claiming extra-revenues due to extra-work to be executed is a skill that, generally, has to be specially acquired. Pinnell (1998) reinforces this argument when he mention that the individual (or the team) responsible for thoroughly analyzing the bid documents aiming to capture and quantify hidden-value during the bid preparation process may be a consultant expert or a team of consultant experts. Whether this incremental investment in hiring skilled consultants and contracting highly specialized firms aiming to supply contractors with more accurate information regarding the volume of work to be performed will be worthwhile constitutes the question to be addressed later on. Our model is thus based on the argument that uncertainty can add value to construction projects through the impact caused on the amount of work to be executed during the project’s life cycle. This argument entails that contractors do not know, before the completion of the project (or, at least, before the job begins),

[2]To the best of our knowledge, this designation was first adopted by Ford et al. (2002) and, in the context of their research, the definition encompasses other sources of hidden-value, rather than just those that may result in additional income.

[3]Our model will just consider a specific type of hidden-value: the one that may result in the creation od additional profit through the execution of more volume of work.

[4]Specialized firms are frequently hired with the purpose of providing the contrator with more accurate information concerning the soil and the under-soil conditions he or she will encounter in the beginning of the construction stage. Not rarely, these studies indicate that the type of foundations defined by the client and included in the bid package is not safe and/or may need to be preceded by a deeper drilling work.

how much volume of work will actually be executed. Hence, uncertainty is present concerning the volume of work, allowing us to designate this specific type of uncertainty, from the contractor's perspective, as "volume uncertainty". Volume uncertainty leads to uncertainty about the project's final value since the execution of additional work implies receiving extra income (or extra revenues) and incurring in extra costs. We will designate, from now on, the difference between these extra revenues and these extra costs as "additional value". Additional value is, therefore, the value that may be generated because there is, at least, a specific source of uncertainty surrounding construction projects that may actually cause such effect. We now proceed to discuss this subject with more detail.

2 Recognizing and Quantifying Hidden-Value: The Concept of Additional Value

In many construction projects value is hidden in the most uncertain portions of the project, as we previously mentioned. After its detection and quantification, hidden-value becomes what we designate as additional value. To fully understand how hidden-value may be detected and properly quantified - and, hence, transformed into additional value - we must first know where hidden-value can be detected, which means we have to understand the nature of its sources.[5] The construction management literature has been dedicating considerable attention to a subject commonly known as "Claims". A construction claim can be defined as "a request by a contractor for compensation over and above the agreed-upon contract amount for additional work or damages supposedly resulting from events that were not included in the initial contract" (Adrian (1993)). This well-known definition implies that contractors can and should ask for a compensation when they execute works that were not considered in the initial contract.[6] Thomas (2001) argued that variations to the work are almost inevitable and Dyer and Kagel (1996) went even further when they stated that - inevitably (sic) - situations arise where clients actually deviate from the original construction scope, which means that, most likely, the initial scope will be increased. These statements strongly sustain our argument that, at least frequently, contractors do end up executing more work than the one deriving from what is established in the tender documents. Consequently, both statements also support the argument that contractors do not know, ex-ante, the precise amount of work they will be executing throughout the whole

[5]The pure detection of hidden-value does not necessarily result in the creation of additional value to the project. The project value will only increase if the execution of the extra volume of work originates a profit. We will discuss this crucial aspect later on.

[6]This definition also implies that there are other sources which may raise more income. However and as we have been stressing, we are only concerned with the ones directly associated with the possible execution of more volume of work.

construction phase. Rooke et al. (2004) categorize construction claims in two different types: (i) proactive claims and (ii) reactive claims. Proactive claims are the ones that can be anticipated and, thus, planned for at the bid preparation stage. On the other hand, reactive claims are the ones that can only be recognized in the course of the project itself, in response to unforeseen events. Even though we are aware that reactive claims may have a substantial impact on the value of the project, we exclude them from our model precisely because they are, by definition, unforeseeable, which means that no acceptable estimate can be drawn. Therefore, our model incorporates estimates for those proactive claims that derive from the presence of uncertainty regarding the volume of work to be performed.[7]

2.1 Sources of Additional Value

There are two sources of uncertainty that may result in claims through the execution of more volume of work than the one directly deriving from the information contained in the bid package. We classify them as being of two different kinds: *(i)* extra quantities and *(ii)* additional orders.

2.1.1 Extra Quantities

Extra quantities occur when the contractor ends up executing, in the field, more quantities of a specific item than the ones specified in the tender documents. As we will see, if the type of contract allows such, the contractor will receive the unit price included in his or her proposal multiplied by the quantities he or she has actually executed and after being measured in the field by the client or the client's agent.[8] Under these contractual conditions, field quantities are the quantities that matter because they are the ones that will generate the income associated with the execution of each task included in the bid package. Ideally, from the client's point of view, field quantities should match the quantities included in the tender documents. However, frequently, discrepancies between the quantities estimated by the client and quantities actually executed in the field are observed. The literature refers that this inaccuracy is mainly due to the poor quality of the tender documents (see, for example, Laryea (2011); Rooke et al. (2004); Akintoye and Fitzgerald (2000)), meaning that the client's estimates are not always accurate and, therefore, tender documents provided to the bidders often contain mistakes. Bearing this in mind,

[7] As we will carefully explain, depending on the contractual arrangements binding the parties, there is a specific type of volume uncertainty which does not necessarily generate more income.

[8] Such fact also implies that the contractor will receive less money if the quantities executed in the field are smaller than the ones specified in the bid documents.

most experienced contractors do not take for granted the accuracy of the information contained in the tender documents regarding the quantities to be performed when they are preparing the bid. On the contrary, if hidden-value is to be captured and quantified - since inaccuracies in the tender documents are likely to occur - mistakes can only be recognized if a proper measurement of all the technical drawings is performed. This is an important aspect we must stress: contractors will only know, with a strong degree of certainty, how many quantities they will be executing during the project's life cycle if a thorough and accurate measurement of all the technical drawings included in the bid package is undertaken. Moreover, as Rooke et al. (2004) stated, pricing a tender involves reading through bills of quantities often several inches thick, meaning that the quantities stated in the must be confronted with the quantities obtained after performing a complete examination of all the drawings provided by the client. Rooke et al. (2004) also argued that, most of the times - especially in the case of non-large contractors - companies do not have experts in this type of highly skilled job or, if they do, the amount of work in hands in a particular moment may imply the need for hiring external experts. This aspect is reinforced by the fact that contractors actually express concern over what they consider to be a short period of time that is normally allowed for bid preparation, as Laryea and Hughes (2008) observed.

2.1.2 Additional Orders

Additional orders, also known as "change orders", refer to a task or a set of tasks the contractor effectively performs during the project's life cycle and that possess a different nature from the ones specified in the bid package. This source of uncertainty that may give rise to additional work and extra profit is, thus, different from the one mentioned before, since change orders are related with varied work which is not of a similar character, or is not carried out under similar conditions than the one contained in the bid package (Davinson (2003)).[9] However, we need to make clear that these tasks may include, for the purposes of their completion, the execution of an item or a set of items that actually were considered in the bill of quantities and previously priced by the bidder, since they were part of the project's initial scope. Hence, when contractors look for mistakes in the tender documents, they do not focus their attention merely in finding discrepancies that may lead to the execution of extra quantities solely associated with the tasks specified in the bid package. Instead, experienced engineers and skilled experts also search for possible tasks, which are likely to be executed and were not specified in the tender documents.

[9] Change orders are also often designated as "increase in scope". The designation acknowledges the fact that the scope of the original project becomes wider, meaning that tha contractor will end up executing a certain number of tasks that were not included in the technical pieces that support the initial contract.

By carefully analyzing all the plans and drawings provided by the client, it is possible to recognize that some parts of the project (or even the project "seen" as a whole) will not be properly completed if only the tasks included in the tender documents are to be performed. Hence, additional orders can and should be considered as a potential source of additional value and our model will consider this argument by assuming that contractors are able to stipulate a high-value estimate and a low-value estimate for the amount of additional orders, and also attribute, to each of these estimates, a probability of occurrence. For the purpose of accurately defining the two estimates, we argue that contractors need to take into account: *i)* the amount of work the additional orders will generate in comparison to the amount established in the original contract; *ii)* the previous experience with the client as well as their history and frequency of placing new orders; *iii)* the bargaining skills of the client throughout the negotiation process.

2.2 Types of Contract

To fully understand the possible impact of the two sources of additional value on the project's final profit, we have to relate each of them with the type of contract that will bind the parties. Construction management literature addresses with more relevance two types of contracts (see, for example, Halpin and Senior (2011); Clough et al. (2000); Woodward (1997)): *(i)* the "unit-price" type of contract and *(ii)* the "lump-sum" type of contract.[10] The unit-price contract allows for flexibility in meeting variations regarding the amount and quantity of work encountered during the construction stage. This means that, when this type of contract is adopted, the project is broken down into work items, which are characterized by units, such as cubic yards, linear and square feet, and piece numbers (Halpin and Senior (2011)). This fact implies that the contractor, during the bid preparation stage, will quote the price by units rather than as a single total contract price. This means that if, for some reason, the contractor effectively executes more quantities of one or more specific items included in the tender documents, he or she will be receiving the amount that results from multiplying the number of units executed by the unit price he or she has included in the bid proposal. If the type of contract enforced is the lump-sum, bidders are asked to price a specific task or item, regardless of the number of units that will actually be executed. Hence, if this type of contract is adopted, contractors will never receive more (less) income for executing more (less) quantities of an item or items clearly specified by the client than those he or she has predicted after analyzing the drawings

[10]Other types of contract are mentioned in the literarure, as the "cost-plus-fee" type and the "cost-reimbursement" type. However, unit-price and, especially, lump-sum contracts are the ones that are most commonly adopted, particularly in the context of bidding competitions.

and other technical documents contained in the bid package. The risk associated with the likelihood of performing more quantities than those that served as the basis for computing the corresponding global price for a specific task (or a group of tasks) is, thus, borne by the contractor. However, the opposite may also occur: contractors might actually perform less quantities in the field than those considered during the bid preparation process, and which served as the basis for establishing the proposed bid price. Hence, and even though this specific type of uncertainty still exists when the parties are bounded by a lump-sum contract, it is not possible to account for its effects during the bid preparation stage since the contractor will only be aware if any additional value is actually raised through this mean after the task or tasks in question are executed, *i.e.*, as the project unfolds. Therefore, this source of uncertainty may affect the project value but can not be quantified before the project is initiated. Being so, in the presence of a lump-sum contract, additional value may only be obtained through the execution of additional orders whereas, if the contract assumes the unit-price type, both sources of uncertainty may create additional value by increasing the volume of work to be performed.[11] Lump-sum contracts are the most common type of contracting, especially in the building sector (Rooke et al. (2004)). In the European Union, current legislation concerning public contracting virtually imposes the lump-sum form, which means that the unit-price type has had small to none application due to the increasing effort european regulators have been exercising with the purpose of transferring the risk associated with possible mistakes (also referred to, in technical language, as “errors and omissions”) encountered in the technical pieces from the client to the contractor. This broad reality has compelled us to consider in our model only one of the two sources of additional value previously described: the additional value that may rise from the execution of additional orders. Thus, we will assume that the lump-sum type of contract is the one that actually binds the parties, thus implying that the possible execution of more quantities in the field than the ones eventually stated in the bid documents will not generate any additional revenues and, consequently, any additional value to the project. This also implies that the costs associated with the possible execution of any extra quantities should be taken into account when determining the amount of constructions costs that will sustain the bid price.

[11]Some projects encopass both types of contracts, which means that some tasks should be priced using unit-prices and otheres applying the lump-sum form. In such cases, both types of uncertainty are present, in diferent parts of the project.

3 How the Detection of Hidden-Value May Lead to Additional Value: Contractor's Opportunistic Bidding Behavior

Construction management literature clearly acknowledges the fact that the construction industry features strong levels of price competitiveness (see, for example, Chao and Liu (2007); Skitmore et al. (2007);Skitmore (2002); Ngai et al. (2002)) which may force bidders to lower their profit margin and, hence, increase the probability of winning the contract (Mohamed et al. (2011)). Consequently, it is not rare to see the winning bid include a near-zero profit margin (Chao and Liu (2007)) or even a price below-cost. This intense competition encountered in bidding processes often leads to "under-pricing", a common phenomenon namely explained by the need for work and penetration strategies (Yiu and Tam (2006); Fayek (1998); Drew and Skitmore (1997)).[12] In fact, contractors realize that bidding low when facing strong competition increases the chance of being selected to execute the project but they are also aware of the opposite: if the profit margin included in their proposals is higher, the probability of getting the contract will be lower. This inverse relationship between the level of the profit margin and the probability of winning the bid is a generally accepted fact both in the construction industry and in the research community (e.g., Christodoulou (2010); Kim and Reinschmidt (2006); Tenah and Coulter (1999); Wallwork (1999)). By adopting Ribeiro et al. (2017) numerical solution, our model incorporates this crucial element and a mathematical expression that respects the inverse relationship between these two variables is adopted.

Detecting hidden-value and executing more volume of work will only result in more value to the project if the difference between the extra revenues and the extra costs of performing the additional tasks is positive, i.e., if contractors do actually generate a profit by executing them, which means that detecting and executing more volume of work than the one directly specified in the tender documents will not necessarily lead to more profit. However, experienced contractors that capture hidden-value ensure themselves that items where extra quantities are likely to be executed will be priced in a way that will lead to an increase in the project value. By applying this practice during the bid preparation stage, contractors increase their probability of winning the bid by sacrificing the profit margin included in the bid proposal, knowing that they may recover (at least) a part of the profit in subsequent change orders or claims, as Tan et al. (2008) observed. This type of behavior is

[12] Under-pricing is not necessarily the same as bidding below-cost; rather, we interpret this concept as the practice of including in the bid price a profit margin lower than the one contractors would include in normal circumstances, *i.e.*, if the levels of price competition in the construction industry were not generally perceived as being pasrticularly intense.

designated in the literature as "Opportunistic Bidding Behavior" (OBB).[13] Thus, following a proactive approach and assuming that time is actually invested in detecting mistakes that may lead to the likely execution of more (less) quantities, contractors will inflate (deflate) the unit price of the items where those mistakes were spotted. Over-charging (under-charging) those items can then be compensated by under-charging (over-charging) the unit-prices of some of the items whose quantities contractors are certain to be accurately measured. Hence, this compensation mechanism allows contractors to maintain the previously defined overall price for executing the quantities specified in the tender documents and, still, leaving room for generating more profit through the likely execution of additional quantities. Despite the fact that this behavior is potentially more effective in the presence of a unit-price form of contract, it may also produce positive effects when the type of contract enforced is the lump-sum. In fact, experienced contractors will most likely inflate prices of items they predict to be present in future additional orders since - and even though additional orders are subject to a specific process of price negotiation - it is likely that they will contain the execution of certain items which were considered in the original contract and, hence, whose price is already established between the parties. In these circumstances, the parties will agree that the unit price for such items will be the same. However, items that are different in their nature from the ones contained in the tender documents become a matter of negotiation between the contractor and the client or the client's agent, as Dyer and Kagel (1996) stated. This means that, unlike what happens with extra quantities, there is no predetermined form of pricing additional orders in its full extension. In fact, contractors do not have a way of predicting, with complete certainty, what price will be established and what profit will be generated if these additional orders are placed by the client. Nevertheless, based on previous experiences and in current market prices, we believe that contractors can actually perform fair estimates on the final revenues to be generated by these additional orders and we also believe that, in the event such orders are placed and the additional work executed, a considerable profit will be made.[14]

The remainder of the present paper unfolds as follows. In Section 4 we introduce the

[13]The literature identifies two different types of OBB: "front loading" and "claim loading" (see, for example Arditi and Chotibhongs (2009); Yiu and Tam (2006)). Front loading consists in over-charging the tasks to be executed in the earlier stages of the project life cycle and compensate such effect by under-charging the tasks to be performed in the later stages. We are not concerned with this type of behavior in our model since it does not derive from any detection of hidden-value.

[14]Dyer and Kagel (1996) conducted a study where a number of general contractors were interviewed. Additional orders were frequently mentioned as being particularly profitable. It is generally accepted in the industry that the negotiation process leading to the price determination of new orders often develops in avery favorable manner to the contractor, manly due to the client's awareness that the decision to switch to another contractor for merely executing those additional tasks will imply incurring in side costs and will also cause time delays.

model's basic numerical solution proposed by Ribeiro et al. (2017), and which will enable us to reach the optimal price if no detection and quantification of hidden-value is considered, *i.e.*, the "optimal base price". We then proceed to Section 5 where the model is fully described. In Section 6, a numerical example is presented, followed by a sensitivity analysis to some of the model's most important parameters. Finally, in Section 7, conclusions and remarks are presented.

4 The Model's Basic Numerical Solution

The model herein suggested uses the same numerical solution proposed by Ribeiro et al. (2017). In their paper, these authors identified a specific real option available to construction companies in the context of bidding competitions: the option to sign the contract and to perform the project by the selected bidder. In a bidding competition, the contractor prepares the bid proposal and submits it until a certain date previously defined by the client. However, the client will only decide which bidder will be invited to sign the contract months later. Consequently, the estimated constructions costs that served as the basis to establish the price included in the bid proposal will most likely vary during this period, *i.e.*, from the moment the bid proposal is closed and delivered to the client until the selected bidder is invited to sign the contract by the client. On the contrary, the price established by the contractor and proposed to the client will remain unchanged during this same period. Thus, the selected bidder will have the right - but not the obligation - to sign the contract and to invest in performing the project. This option constitutes a real option and, as stated by the option pricing theory, does have value and Ribeiro et al. (2017) evalated it with the final purpose of reaching an optimal price. According to these authors, the optimal price will be the one corresponding to the highest value of the option to invest, weighted by the probability of winning the bid, since the option can only be exercised by the selected bidder. Ribeiro et al. (2017) constructed a mathematical relationship linking the level of the profit margin (or the "mark-up", as it is commonly designated in construction parlance) with the probability of winning the contract.

Being so, the numerical solution presented by Ribeiro et al. (2017) has two components: *(i)* the value of the option to sign the contract and, consequently, to invest in executing the project, firstly in the absence of any penalty costs and, secondly, considering that penalty costs are enforced by the client should the selected bidder refuses to enter into contract; *(ii)* the probability of winning the bid. We present and briefly discuss each of them below.

4.1 The Value of Option to Sign the Contract and Invest in Performing the Project

The value of the option to sign the contract and perform the project is computed by Ribeiro et al. (2017) considering two possible scenarios: first, the vakue of the option in question is computed not considering the existence of any penalty costs in the event the selected bidder refuses to enter into contract at the moment the contract needs to be signed. However, as these authors stressed, in some legal environments, a compensation may be claimed by the client should the selected bidder decides no to sign the contract. For example, and according to Halpin and Senior (2011), in the United States contractors are free to withdraw their bids without incurring in any penalties if that happens prior to the ending of the bidding period. However, if a contractor decides to withdraw the bid after that moment - and assuming that he or she is the selected bidder - a penalty equal to the difference between the second best proposal and the selected bid is legally imposed, even if the contract has not yet been signed. Thse authors write that "this may occur in the event that the selected bidder realizes that he or she has underbid the project and that pursuing the work will result in a financial loss" (p.44).[15] In these circumstances, the client may exercise the legal right of receiving the difference between the two bid prices.

Let P denote the price included in the bid proposal and K_t the expected amount for the construction costs computed during the bid preparation stage, solely for the execution of the tasks included in the bid package.[16] The bid price P is established by the contractor and remains unchanged after being determined. It is, therefore, and as we mentioned previously, constant throughout the life of the option. This is the price that the company will charge if he or she happens to be the selected bidder and if the project is executed. However, the construction costs change overtime, being a very relevant source of uncertainty for the contractor. We should make clear that the expected construction costs are observable and also that their exact value will only be known after the project is completed. We model the expected amount of construction costs, K_t, as a geometric Brownian motion, which is given by the following equation:

$$dK_t = \alpha K_t dt + \sigma K_t dz \tag{1}$$

where α is the drift parameter (the expected rate of increase of K_t), σ is the standard

[15] We believe the authors use the word "underbid" to express the fact that the selected bidder may realize, in the day the contract has to be signed, that the expected construction costs are greater than the bid price.

[16] Contractors will most likely incur in additional costs due to the execution of more volume of work. However, these extra-costs will be considered for the purpose of estimating the additional profit to be generated and, hence, the definition of K_t as being the base construction costs will remain unaffected.

deviation (the volatility parameter) and dz is the increment of a standard Wiener process. The parameters α and σ can be estimated by using market data, as they represent the expected future behavior of the investment costs.

In the presence of penalty costs, the payoff (at maturity) of the option to sign the contract in a flexible context is given by[17]:

$$Max[P-(1-g)K_T;0]-gK_T \tag{2}$$

For the purposes of this research piece, we wil assume that penalty costs are present and designated by g, and estimated as being a percentage of the expected construction costs at the bid preparation stage.[18] This implies contractors possess historical information enabiling them to produce an average percentage expressing the historical numerical difference between his or her proposal and the second best proposal. Being so, contractors may set the appropriate g for a given bidding competition, as a function of K_t , the expected amount of construction costs necessary to perform the project in hand.

The value of the option to sign the contract for the selected bidder, at any time $t < T$, can be computed *a la* Black and Scholes (1973) as follows (where P is fixed and K is stochastic):

$$F(P,K_t) = Pe^{-r(T-t)}N(d_1)-(1-g)K_tN(d_2)-gK_t \tag{3}$$

and:

$$d_1 = \frac{ln[P/((1-g)K_t)]-(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{4}$$

where:

$$d_2 = d_1-(\sigma\sqrt{T-t}) \tag{5}$$

The terms $N(d_1)$ and $N(d_2)$ represent the standard normal cumulative distribution function for the values resulting from equations (4) and (5), respectively, and r is the risk-free interest rate.

[17]For a thorough explanation of how the presence of penalty costs affects the selected bidder's payoff, please refer to Ribeiro et al. (2017).

[18]If penalty costs are absent, then $g = 0$ and equation (2) will simply be: $Max[P-K_T;0]$, *i.e.*, equal to the payoff of an european *call* option.

4.2 The Probability of Winning the Bid

The option to sign the contract and invest in executing the project is only available to the selected bidder and equation (3) allows us to compute its value. However, the value of this option for a given contractor must depend also on the probability of winning the bid. So it is necessary to set an appropriate winning function that captures this probability. Ribeiro et al. (2017) propose an inverse relationship linking the mark-up ratio, P/K_t and the probability of winning the bid, $W(P,K_t)$, which is given by the following equation: [19]

$$W(P,K_t) \quad = \quad e^{-b(P/K_t)^n} \tag{6}$$

where 'n' and 'b' are parameters that should be used to calibrate the expression linking the mark-up ratio and the probability of winning the contract in order to best reflect each contractor specific circumstances, as we previously argued. Parameter '*n'* is responsible for shaping the winning function's concavity and convexity. On the other hand, parameter '*b'* sets the probability of winning the bid with a zero-profit margin. Thus, in a bidding process where the level of of competition is perceived to be extremely high, the construction manager should establish a value for parameter 'b' that reflects such perception, say $b = ln(1/0.2)$, which means that the manager estimates the probabibility of winning the contract with a zero mark-up as being only 20%. In general terms, parameters 'b' and 'n' may also capture the results of data compiled by managers from past bidding competitions, thus allowing them to design their winning function accordingly. Also, with a specific calibration, it is possible to set parameters 'n' and 'b' in order to capture a linear relationship between the two variables.

4.3 The Base Price

Let P denote the base price. Considering the formula that computes the value of the option to perform the project and execute the project in the presence of penalty costs, the model's outcome is the solution for the following maximization problem:

$$V(P,K_t) \quad = \max_P \quad \left\{[Pe^{-r(T-t)}N(d_1) - (1-g)K_tN(d_2) - gK_t][e^{-b(P/K_t)^n}]\right\} \tag{7}$$

[19]Ribeiro et al. (2017) present a solution where the price is normalized in relation to the construction costs (hence, the mark-up ratio is the variable, not the price). These authors also point-out that they could have used the price instead of the mark-up ratio and the results will be identical.

The value of P^* , the optimal base price, can be found by solving numerically the following equation:

$$e^{-r(T-t)}N(d_1)e^{-b(P/K_t)^n} + [Pe^{-r(T-t)}N(d_1) - (1-g)K_tN(d_2) - gK_t]\frac{-be^{-b\left(\frac{P}{K_t}\right)^n} n\left(\frac{P}{K_t}\right)^n}{P} = 0 \quad (8)$$

The optimal base price P^* is the optimal price computed by the contractor in the case that no 'hidden-value' is captured and measured during the bid preparation stage and is the solution for the maximization problem given by equation (7). It is, therefore, the optimal bid price suggested by Ribeiro et al. (2017) in the presence of penalty costs. We now proceed to present the model that incorporates the effects of capturing and quantifying 'hidden-value' during the bid preparation stage on the optimal bid price.

5 The Model

5.1 Assumptions

We will assume that *(i)* each bidder decides what price to bid without engaging in any kind of interaction or contact with other bid participants; *(ii)* each bidder prepares his or her proposal simultaneously with the other competitors; *(iii)* each bidder presents a single-sealed proposal to the client; *(iv)* each bidder has access to the information contained in the bid package, allowing him or her to establish the base price, determined applying the maximization problem established by Ribeiro et al. (2017) and expressed by equation (7); *(v)* the selected bidder will only decide if he or she is going to perform the project when the contract has to be signed and not before that date; *(vi)* the parties are bound by a lump-sum contract, which means that no additional income is generated if extra quantities are executed; *(vii)* by only using the skills of their own experienced staff, contractors are able to stipulate a high-value estimate and a low-value estimate for the expected profit to be generated through the execution of additional orders and also to attribute a probability of occurrence to each of them, during the bid preparation stage; *(viii)* once the true estimate is known, contractors will adjust the price (extra revenues) during the negotiation process to compensate for any variations that may occur in the estimated extra costs, which means that any changes observed in the necessary costs to perform the additional orders will lead to an adjustment in the price requested to the client, with the purpose of maintaining the expected profit at the level previously established during the bid preparation period.

5.2 Model Description

5.2.1 Introduction

The present model is motivated by the fact that, in most construction projects, the volume of work to be executed is not known with precision during the bid preparation stage. Hence, uncertainty is present concerning the level of profit to be generated. Even though we have identified two different sources of volume uncertainty, we will only focus on the one deriving from the possible execution of additional orders to be placed by the client, since we are assuming that the parties are bound by a lump-sum contract - thus preventing contractors from generating extra revenues by merely executing extra quantities of items included in the bid package. As previously mentioned, we will consider that experienced contractors are be able to stipulate a high-value estimate and a low-value estimate to the additional orders and to attribute a probability of occurrence to each of them just by undertaking a preliminary assessment of the tender documents, which means that this goal can be achieved without the need to incur in any additional costs associated with hiring skilled professionals and contracting specialized firms, *i.e.*, without any incremental investment in human capital and technology. For the sake of simplicity, we will refer to the mere allocation of working time of the persons possessing the necessary skills to perform these tasks (*e.g.*, engineers and estimators) as "non-incremental investment". Let C_1 designate the level of this non-incremental investment that contractors will undertake using only the skills of their own experienced staff. By investing the amount C_1 , contractors will thus *(i)* define a high-value estimate and a low-value estimate for the price (revenues) to be obtained through the execution of additional orders; *(ii)* stipulate a high-value estimate and a low-value estimate for the necessary costs to successfully perform these orders; *(iii)* attribute a probability of occurrence to each of the estimates. Therefore, by investing the amount C_1, contractors will be establishing a discrete-time stochastic variable, designated as "additional value", with two possible outcomes, and affecting a probability of occurrence to each of them.

5.2.2 The Impact of Non-Incremental Investment

The optimal base price, P^* represents the amount of income to be received due to the execution of the volume of work included in the bid package: this is the price resulting from equation (7) suggested by Ribeiro et al. (2017) and - again - we stress that this is the optimal price contractors should include in their proposals if no detection and quantification of any additional value deriving from the execution of additional orders is undertaken. By investing the amount C_1, contractors will most likely detect and quantify hidden-value, which may result in the creation of additional value to the project. Hence, we first need to consider the additional income that will be received, assuming that additional orders will be executed during the project's life cycle and, secondly, the necessary costs to successfully perform the additional work. Being so, let p_A represent the additional income (extra revenues) that derives from the possible execution of additional orders, and k_A the amount of costs the contractor will have to incur in order to perform these new orders. Being so, let π represent the amount of profit (or additional value) generated by executing the additional orders, *i.e.*, the difference between p_A and k_A. Also, let *(i)* p_A^H designate the high-value estimate for the revenues associated with the execution of the additional orders; *(ii)* p_A^L designate the low-value estimate for such revenues; *(iii)* k_A^H designate the high-value estimate for the costs associated with the execution of the additional orders; (*iv*) k_A^L designate the low-value estimate for such costs; (*v*) θ represent the probability associated with p_A^H and k_A^H ; hence $(1-\theta)$ is the probability associated with p_A^L and k_A^L. Finally, let π^H and π^L denote the additional profit for the high-value and the low-value estimates, respectively. π^H and π^L will be given by the following equations:

$$\pi^H = p_A^H - k_A^H \tag{9}$$

$$\pi^L = p_A^L - k_A^L \tag{10}$$

Thus, the expected value for the additional profit, $E(\pi)$ will be given by equation (11):

$$E(\pi) = \pi^H\theta + \pi^L(1-\theta) \tag{11}$$

Let P_1^* designate the price in the present conditions, *i.e.* the price that incorporates the effect of the expected value for the additional profit, $E(\pi)$, given by equation (11). Hence, we adapt equation (7) with the purpose of incorporating the effects caused by the expected value for the additional profit. Being so, P_1^* will be the outcome for the following maximization problem:

$$V(P_1,K_t) = \underset{P_1}{max}\left\{[((P_1+E(\pi))e^{-r(T-t)}N(d_1)-(1-g)K_tN(d_2)-gK_t][e^{-b(P_1/K_t)^n}]\right\} \tag{12}$$

where:

$$d_1 = \frac{ln[((P_1+E(\pi))/((1-g)K_t)]-(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{13}$$

and:

$$d_2 \;=\; d_1 - (\sigma\sqrt{T-t}) \tag{14}$$

The value of the option to invest, under these new conditions, will be given by the equation below:

$$V(P_1^*,K_t) = \Big\{[(P_1^* + E(\pi))e^{-r(T-t)}N(d_1) - (1-g)K_tN(d_2) - gK_t][e^{-b(P_1^*/K_t)^n}]\Big\} \tag{15}$$

The optimal price, P_1^* that results from the maximization problem given by equation (12) is smaller than the one resulting from the maximization problem given by equation (7), *i.e.*, the optimal base price, P^* since the former is the optimal price in the absence of any recognition and quantification of hidden-value generating more profit through the execution of additional orders, whereas the latter reflects the optimal price considering the expected impact of the additional orders to be performed, at this stage, by investing the amount C_1.[20] Being so, P_1^* is the price consruction managers should include in their bid proposals because it is the optimal price if no incremental investment is undertaken. Also, we should stress that just by investing the amount C_1, construction managers will always produce a more competitive bid price, provided that some hidden-value leading to the generation of additional profit through the execution of additional orders has actually been captured and quantified.

5.2.3 The Impact of the Incremental Investment in Human Capital and Technology

As we mentioned previously, construction companies often have to invest in human capital and technology and, hence, hire skilled technicians and/or highly specialized firms possessing the necessary know-how and technology to perform specific studies, whose purpose is to supply managers with more accurate information concerning the project in hand, during the bid preparation stage. Let C_2 denote the value of this incremental investment, which will allow the contractor to eliminate the uncertainty concerning the true value of the additional work to be performed and the extra profit to be generated through the execution of such additional work. Hence, after investing the amount C_2, the contractor may face two different scenarios, since this investment will reveal if the true value is the high estimate or the low estimate. Therefore, we first need to determine the optimal price and the corresponding value of the option to invest, for each of the scenarios.

[20] Assuming that the expected value for the additional profit, E (π) is actually positive.

5.2.3.1. The Optimal Price and the Value of the Option to Invest Considering the High-Value Estimate

If the investment in C_2 reveals that the true value for the additional profit, $\pi$is given by the high-estimate, then the optimal price in these conditions, $P_2^{*^H}$ will be the outcome of the maximization problem given by the following equation:

$$V(P_2^H, K_t) = \max_{P_2^H} \left\{ [((P_2^H + \pi^H)e^{-r(T-t)}N(d_1) - (1-g)K_t N(d_2) - gK_t][e^{-b(P_2^H/K_t)^n}] \right\} \tag{16}$$

where:

$$d_1 = \frac{ln[(P_2^H + \pi^H)/((1-g)K_t)] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{17}$$

and:

$$d_2 = d_1 - (\sigma\sqrt{T-t}) \tag{18}$$

The value of the option to invest, assuming the true value for the additional profit equals the high-value estimate, is given by equation (19):

$$V(P_2^{*^H}, K_t) = \left\{ [(P_2^{*^H} + \pi^H)e^{-r(T-t)}N(d_1) - (1-g)K_t N(d_2) - gK_t][e^{-b(P_2^{*^H}/K_t)^n}] \right\} \tag{19}$$

5.2.3.2. The Optimal Price and the Value of the Option to Invest Considering the Low-Value Estimate

However, if the investment in C_2 reveals that the true value for the additional profit, π is given by the low-estimate, then the optimal price in these conditions, $P_2^{*^L}$ will be the outcome of the maximization problem given by the following equation:

$$V(P_2^L, K_t) = \max_{P_2^L} \left\{ [((P_2^L + \pi^L)e^{-r(T-t)}N(d_1) - (1-g)K_t N(d_2) - gK_t][e^{-b(P_2^L/K_t)^n}] \right\} \tag{20}$$

where:

$$d_1 = \frac{ln[(P_2^L + \pi^L)/((1-g)K_t)] - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{21}$$

and:

$$d_2 \quad = \quad d_1 - (\sigma\sqrt{T-t}) \tag{22}$$

Similarly, the value of the option to invest, assuming that the true value for the additional profit equals the low-value estimate, is given by equation (23) below:

$$V(P_2^{*^L}, K_t) = \left\{ [(P_2^{*^L} + \pi^L)e^{-r(T-t)}N(d_1) - (1-g)K_t N(d_2) - gK_t][e^{-b(P_2^{*^L}/K_t)^n}] \right\} \tag{23}$$

5.2.4 The Threshold Value for the Incremental Investment

Let θ denote the probability associated with the high-value estimate for the additional profit and $(1-\theta)$ the probability associated with the corresponding low-value estimate. By weighting the outcome of equation (19) with the probability associated with the high-value estimate, θ and weighting the outcome of equation (23) with the probability associated with the low-value estimate, $(1-\theta)$, we are able to reach the value of the option to invest considering the two scenarios, $V(P_2^*, K_t)$:

$$V(P_2^*, K_t) = \theta V(P_2^{*^H}, K_t) + (1-\theta)V(P_2^{*^L}, K_t) \tag{24}$$

The outcome of equation (15) is the value of the option to invest considering solely the effects produced by the non-incremental investment, C_1, *i.e.,* $V(P_1^*, K_t)$. Equation (24) determines the value of the option considering the effects of the incremental investment in human capital and technology, C_2 considering both scenarios together, each weighted by the corresponding probability of ocurrence. Hence, the difference between the outcome of equation (24) and the outcome of equation (15) is the exact amount of the incremental investment, C_2 below which any level of incremental investment will add value to the project. Let C_2^* denote this threshold value for the incremenal investment. C_2^* will thus be given by the following equation:

$$C_2^* = V(P_2^*, K_t) - V(P_1^*, K_t) \tag{25}$$

and the ratio between the incremental investment threshold and the base construction costs computed during the bid preparation stage is as follows:

$$RC_2^* = \frac{C_2^*}{K_t} \tag{26}$$

6 Numerical Example

6.1 The Base Case

The following table includes information about the inputs used in the present numerical example:

Table 1: inputs: description and values

Inputs	Description	Values
K_t	base construction costs	\$100,000,000
σ	standard deviation	0.25
r	risk-free interest rate	0.01
$T-t$	"time to expiration"	0.5 (years)
n	calibration parameter for the function $W(P,K)$	10
b	calibration parameter for the function $W(P,K)$	ln(1/0.5)
g	penalty costs	0.02
p_A^H	high-value estimate for the aditional revenues	\$30,000,000
k_A^H	high-value estimate for the aditional costs	\$15,000,000
p_A^L	low-value estimate for the aditional revenues	\$9,000,000
k_A^L	low-value estimate for the aditional costs	\$6,000,000
θ	probability of ocurrence of the high-value estimate	0.5
$(1-\theta)$	probability of ocurrence of the low-value estimate	0.5

Using the inputs listed above and applying the model described in Section (5), we have reached the following outputs:

Table 2: outputs: description, values and corresponding equations

Inputs	Description	Values	Equations
P^*	optimal base price	\$101,942,247	(7)
π^H	addtional profit considering the high-value estimate	\$15,000,000	(9)
π^L	addtional profit considering the high-value estimate	\$3,000,000	(10)
$\mathrm{E}(\pi)$	expected value for the additional profit	\$9,000,000	(11)
P_1^*	optimal price after investing C_1	\$99,000,845	(12)
$V(P_1^*,K_t)$	option value after investing C_1	\$6,091,160	(15)
P_2^{*H}	optimal price considering the high-value estimate	\$97,250,537	(16)
$V(P_2^{*H},K_t)$	option value considering the high-value estimate	\$8,650,010	(19)
P_2^{*L}	optimal price considering the low-value estimate	\$100,911,754	(20)
$V(P_2^{*L},K_t)$	option value considering the low-value estimate	\$4,025,678	(23)
$V(P_2^*,K_t)$	option value considering both estimates	\$6,337,844	(24)
C_2^*	incremental investment threshold value	\$246,684	(25)
$R_{C_2^*}$	investment threshold / base construction costs	0.247%	(26)

6.2 Sensitivity Analysis

6.2.1 Is There a Scale-Effect?

Assuming that the project dimension is given by the value of the expected base construction costs, K_t, we performed a sensitivity analysis with the purpose of verifying if a scale-effect is present, *i.e.*, if the investment threshold ratio, $R_{C_2^*}$ assumes different values in response to variations in the amount of the base construction costs. We defined two alternative scenarios, where *(i)* the amount of the base construction costs is twice as great and four times as great as in the base case, *i.e.*, equal to \$200,000,000 and \$400,000,000; *(ii)* the level for the high-value estimate and the low-value estimate of the additional profit respects this same proportion; and *(iii)* the corresponding probabilities of occurrence remain unchanged. We reached the following results for the incremental investment threshold value, C_2^* and for the incremental investment threshold ratio, $R_{C_2^*}$:

Table 3: sensitivity analysis: scale-effect
(for: $\sigma = 0.25$; $T-t = 0.5; n = 10; b = ln(1/0.5)$; $g = 0.02$*)*

	Base Case	**Alternative Scenario 1**	**Alternative Scenario 2**
K_t	\$100,000,000	\$200,000,000	\$400,000,000
π^H	\$15,000,000	\$30,000,000	\$60,000,000
π^L	\$3,000,000	\$6,000,000	\$12,000,000
(θ)	50%	50%	50%
$(1-\theta)$	50%	50%	50%
E(π)	\$9,000,000	\$18,000,000	\$36,000,000
C_2^*	\$246,684	\$493,368	\$986,736
$R_{C_2^*}$	**0.247%**	**0.247%**	**0.247%**

The results included in Table 3 clearly demonstrate that the incremental investment threshold value is proportional to the amount of the base construction costs, which means that no scale-effect is present. In fact, for the three dimensions considered, the ratio between the incremental investment threshold and the base construction costs remains constant and equal to 0.247%, which means that there is a linear relationship between them.

6.2.2 The Impact of Variations in the Probabilities Associated with the High/Low-Value Estimates

The results concerning the impact of considering different levels for the probabilities associated with the high-value and the low-value estimates on the model's outcome are included in Table 4:

Table 4: sensitivity analysis: probabilities associated with high/low value estimates (for: $K_t = \$100,000,000$; $\sigma = 0.25$; $T - t = 0.5; n = 10; b = ln(1/0.5)$; $g = 0.02$)

θ	$(1-\theta)$	$E(\pi)$	$V(P_1^*, K_t)$	$V(P_2^*, K_t)$	C_2^*	$R_{C_2^*}$
99%	1%	\$14,880,000	\$8,594,122	\$8,603,767	\$9,645	0.01%
90%	10%	\$13,880,000	\$8,009,638	\$8,187,667	\$88,029	0.09%
80%	20%	\$12,600,000	\$7,568,344	\$7,725,143	\$156,799	0.16%
70%	30%	\$11,400,000	\$7,056,369	\$7,262,710	\$206,341	0.21%
60%	40%	\$10,200,000	\$6,563,919	\$6,800,277	\$236,358	0.24%
50%	**50%**	**\$9,000,000**	**\$6,091,160**	**\$6,337,844**	**\$246,684**	**0.247%**
40%	60%	\$7,800,000	\$5,638,219	\$5,875,410	\$237,191	0.24%
30%	70%	\$6,600,000	\$5,205,182	\$5,412,977	\$207,795	0.241%
20%	80%	\$5,400,000	\$4,792,090	\$4,950,544	\$158,453	0.16%
10%	90%	\$4,200,000	\$4,398,939	\$4,488,110	\$89,171	0.09%
1%	99%	\$3,120,000	\$4,062,110	\$4,071,920	\$9,810	0.01%

The results included in Table 4 clearly show that, the closer the probabilities are to the upper limit or the lower limit, the smaller is the investment threshold value. The explanation resides in the fact that, the closer the probabilities are to 100% or to 0%, the lower is the uncertainty regarding which will be the true value, meaning that the incremental investment assumes now a lower importance in resolving this uncertainty. The two more extreme scenarios clearly reflect this: when parameter θ equals 99% or 1%, the investment threshold assumes very low values ($C_2^* = \$9,645$ and $\$9,810$, respectively). On the contrary, as probabilities tend to 50%, the higher is the threshold value, C_2^*. Thus, the incremental investment threshold reaches the maximum value when the level of uncertainty is the highest, *i.e.*, when the probabilities associated with the two estimates are the same.

6.2.3 The Impact of Variations in the Difference Between the Two Estimates

Table 5 includes values concerning three different scenarios and the results reached by changing the difference between the high-value and low-value estimates but assuming that both the expected profit, $E(\pi)$ and the probabilities of occurrence, θ and $(1-\theta)$ remain unchanged.

Table 5: sensitivity analysis: difference between the high and the low-value estimates

(for: $\sigma = 0.25$; $T - t = 0.5$; $n = 10$; $b = ln(1/0.5)$; $g = 0.02$)

	Base Case	**Alternative Scenario 1**	**Alternative Scenario 12**
K_t	\$100,000,000	\$100,000,000	\$100,000,000
π^H	\$15,000,000	\$17,000,000	\$13,000,000
π^L	\$3,000,000	\$1,000,000	\$5,000,000
$(\pi^H$-$\pi^L)$	**\$12,000,000**	**\$16,000,000**	**\$8,000,000**
θ	50%	50%	50%
E(π)	\$9,000,000	\$9,000,000	\$9,000,000
$V(P_1^*, K_t)$	\$6,091,160	\$6,091,160	\$6,091,160
$V(P_2^*, K_t)$	\$6,337,844	\$6,528,210	\$6,201,066
C_2^*	**\$246,684**	**\$437,050**	**\$109,906**
$R_{C_2^*}$	**0.247%**	**0.437%**	**0.110%**

In the alternative scenario 1, the difference between the two estimates is greater than in the base case. The results show that, when this difference increases from \$12,000,000 (the difference in the base case) to \$16;000; 000, the incremental investment threshold also increases (from \$246,684 to \$437,050). The explanation resides in the fact that contractors face more uncertainty concerning which of the two estimates will become the true value and, hence, the incremental investment assumes a higher importance in resolving such uncertainty. In fact, the corresponding threshold value is greater since the increase in the value of the option to invest considering both estimates, $V(P_2^*, K_t)$ assumes now a higher value, whereas the value of the option to invest considering solely the effects of the non-incremental investment, $V(P_1^*, K_t)$ remains unchanged. On the contrary, for a smaller difference between the two estimates (\$8,000,000), as the results reached for the alternative scenario 2 clearly reflect, the investment threshold is smaller: \$109,906, compared to \$246,684 in the base case. The level of uncertainty associated with the two estimates is now lower and this lower level of uncertainty is reflected in the value of option to invest considering both estimates, $V(P_2^*, K_t)$.

The value of this option, $V(P_2^*, K_t)$ is, in the alternative scenario 2, smaller than in the other two cases and, thus, closer to the value of the option to invest considering only the investment in C_1, $V(P_1^*, K_t)$ whose value does not depend upon the differences between the two estimates as the expected value for the additional profit, E(π) remains unchanged. We thus conclude that, the higher (lower) the difference between the high-value estimate and the low-value estimate, the higher (lower) will be the uncertainty concerning which estimate will become the true value, and the higher (lower) will be the value of the option to invest considering the two estimates, $V(P_2^*, K_t)$. Consequently - since the value of the option to invest considering only the effects of the non-incremental investment, $V(P_1^*, K_t)$ remains unchanged - the greater (smaller) will be the value for investment threshold, C_2^*

and the greater (smaller) will be the value of the ratio $R_{C_2^*}$.

7 Conclusion and Remarks

Several types of uncertainty surround construction projects and construction managers should proactively manage the effects they may produce in the project value. We approached a specific type of uncertainty and designated it as “volume uncertainty”. This type of uncertainty is critical since, at least frequently, managers do not know with precision the amount of work they will be executing throughout the project’s life cycle and, consequently, the expected final profit the project will generate. To assess the impact of volume uncertainty on the value of the project, we defined a discrete-time stochastic variable and designated it as “additional value”. Additional value is the value that is hidden in the the most uncertain parts of the project and, in the context of the present research, is defined as the one that does not derive from merely executing the tasks specified in the bid package. To capture and quantify this type of value, construction companies need to invest. Initially, by merely applying the skills of his or her own experienced staff, construction managers are able to define a high-value estimate and a low-value estimate for the additional profit and to stipulate a probability of occurrence to each of the estimates. Based on the numerical solution established by Ribeiro et al. (2017), we suggested a model which determines that managers will produce a more competitive bid even if no incremental investment is undertaken, provided that some hidden-value is captured and quantified during the bid preparation stage. However, in order to resolve the uncertainty concerning which of the two estimates will become the true value for the expected additional profit, contractors often need to invest in human capital and 63 technology and, thus, hire specialized firms and highly skilled professionals. The model’s outcome is the threshold value for this incremental investment. Therefore, managers may use a simple decision rule: hire external services with the purpose of eliminating the uncertainty concerning which of the two estimates previously established is the true value, provided that the cost of this incremental investment in human capital and technology does not exceed the threshold value previously determined. Any amount paid for external services, which is lower than the threshold value, will lead to an increase the project value and, the lower this cost, the higher will be the increase in the project value. On the contrary, if the amount actually invested exceeds the predetermined threshold value, the value of the project will be reduced. The model also determines the optimal bid price in the case no incremental investment in human capital and technology is undertaken, in the case the true value reached by undertaking the incremental investment equals the high-estimate for the additional value and also in the case the true value equals the low-estimate for the additional value, both previously stipulated. Sensitivity analysis showed that no scale-effect is present in the model since the incremental investment threshold value responds linearly to variations in the project dimension. Sensitivity analysis also showed that, the closer to 50% is the probability of occurrence associated with the estimates, the greater the threshold value is since undertaking the incremental investment assumes a higher importance due to the presence of higher levels of uncertainty concerning which of the two estimates will become the true value. Finally, sensitivity analysis performed to the difference between the two estimates established for the additional value demonstrated that, the greater the difference between the two estimates the higher the level of uncertainty concerning which of them will become the true value. As a result, the incremental investment assumes a greater importance in eliminating this uncertainty and this greater importance is reflected in a higher threshold value for the incremental investment. On the contrary, if the difference between the two estimates is smaller, the level of uncertainty present is lower, which means that the incremental investment assumes now a smaller importance in resolving this uncertainty and, as a consequence, the incremental investment threshold assumes a lower value.

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